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## EXOSPHERIC PERTURBATIONS BY RADIATION PRESSURE.

## II: SOLUTION FOR ORBITS IN THE ECLIPTIC PLANE

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## ABSTRACT

An earlier paper gave solutions for the mean time rates of change of orbital elements of satellite atoms in an exosphere influenced by solar radiation pressure. Each element was assumed to behave independently. Here the instantaneous rates of change for three elements ( $e$ ,  $\Omega$ , and  $\phi = \omega + \Omega$ ) are integrated simultaneously for the case of the inclination  $i = 0$ . The results (a) confirm the validity of using mean rates when the orbits are tightly bound to the planet and (b) serve as examples to be reproduced by the complicated numerical solutions required for arbitrary inclination. Strongly bound hydrogen atoms escaping from Earth due to radiation pressure do not seem a likely cause of the geotail extending in the anti-sun direction. Instead, radiation pressure will cause those particles' orbits to deteriorate into the Earth's atmosphere. Whether loosely bound H atoms are plentiful enough to create the geotail depends on their source function versus  $r$ ; that question is beyond the scope of this paper.

## INTRODUCTION

In an earlier paper (Chamberlain, 1979, hereinafter referred to as "Paper I") we developed equations for the mean rates of change of the orbital elements of satellite particles in a planetary exosphere that is subjected to solar radiation pressure. The results showed a surprising tendency for the perigees of direct orbits to lock into stable positions westward of the planet as seen from the sun (and eastward for retrograde orbits). These stable positions of perigee are also close to the positions where perigee is most rapidly lowered, vacating the orbits (Paper I, Fig. 4).

It would thus appear that satellite orbits should be efficiently depleted in time scales of the order of [Paper I, Eq. (17)]

$$\begin{aligned} T_0 &\equiv (\mu/a)^{1/2}/f \\ &= 1.055 \times 10^6 / (a/R_E)^{1/2} \text{ sec,} \end{aligned}$$

where  $\mu = GM_E$ ,  $f$  is the acceleration due to radiation pressure, and the orbital notation is conventional and follows that of Paper I. To pursue this matter further, we have investigated the simultaneous solution of the elements  $\phi_s$  (longitude of perigee from the sun),  $e$  (eccentricity) and  $\Omega_s$  (longitude from the sun of the ascending node). In these solutions we set the inclination  $i = 0$ . For the last two elements we will use mean values. The mean rate of change of the semi-major axis vanishes:  $\langle da/dt \rangle = 0$ . And the time of perigee passage,  $\tau$ , offers special problems that make it desirable to adopt its mean value.

## INSTANTANEOUS AND MEAN RATE EQUATIONS

The equations for perturbed elements are given in terms of the external force components by Burns (1976), Burns et al. (1979), and in Paper I. In Paper I, by the way, the value of  $\underline{de/dt}$  is incomplete, since it was derived by holding  $\underline{a} = \text{const.}$ , in anticipation of averaging over the orbit. In reality  $\underline{a}$  varies over a perturbed orbit (only its mean time derivative vanishes) and the correct Eq. (10) of Paper I is

$$\begin{aligned} de/dt = [a(1 - e^2)/\mu]^{1/2} [ & - f_T(1 - e^2)/e(1 + 3 \cos v) \\ & + f_R \sin v + f_T(1 + e \cos v)/e ]. \end{aligned} \quad (2)$$

The error does not affect the derived mean value,  $\langle de/dt \rangle$ , or any other results of Paper I.

To integrate these rate equations, the force components must be expressed in terms of the particle's position in space [Paper I, Eqs. (2), (6), and (7)]. The instantaneous equations are then, for the semi-major axis,

$$\begin{aligned} da/dt = 2f[a^3/\mu(1 - e^2)]^{1/2} [ & \cos \Omega_s \sin (\omega + v) \\ & + \sin \Omega_s \cos (\omega + v) \cos i + e \sin \Omega_s \cos \omega \cos i \\ & + e \cos \Omega_s \sin \omega ], \end{aligned} \quad (3)$$

for the eccentricity,

$$\begin{aligned}
de/dt = f[a(1 - e^2)/\mu]^{1/2} \{ & - [(1 - e^2)/e(1 + e \cos \omega)] [\cos \Omega_s \sin (\omega + v) \\
& + \sin \Omega_s \cos (\omega + v) \cos i] - \sin v [\cos \Omega_s \cos (\omega + v) \\
& - \sin \Omega_s \sin (\omega + v) \cos i] + [(1 + e \cos v)/e] \times \\
& [\cos \Omega_s \sin (\omega + v) + \sin \Omega_s \cos (\omega + v) \cos i] \}, \quad (4)
\end{aligned}$$

for the longitude of the ascending node,

$$d\Omega_s/dt = - f[a(1 - e^2)/\mu]^{1/2} \sin \Omega_s \sin (\omega + v)/(1 + e \cos v), \quad (5)$$

for the inclination,

$$\begin{aligned}
di/dt = - f[a(1 - e^2)/\mu]^{1/2} \sin \Omega_s \sin i \cos (\omega + v)/ \\
(1 + e \cos v), \quad (6)
\end{aligned}$$

for the argument of perigee,

$$\begin{aligned}
d\omega/dt + \cos i \, d\Omega_s/dt = f[a(1 - e^2)/\mu e^2]^{1/2} \times \\
[\cos \Omega_s \cos (\omega + v) \cos v - \sin \Omega_s \sin (\omega + v) \cos i \cos v \\
+ \cos \Omega_s \sin (\omega + v) \sin v (2 + e \cos v)/(1 + e \cos v) \\
+ \sin \Omega_s \cos (\omega + v) \cos i \sin v (2 + e \cos v)/(1 + e \cos v)], \quad (7)
\end{aligned}$$

and finally (Burns, 1976) for the rate of change of the time of perigee passage,

$$\begin{aligned}
 d\tau/dt = & f\{3(\tau - t)[a/\mu(1 - e^2)]^{1/2} e \sin v - [a^2(1 - e^2)/\mu e][\cos v - \\
 & 2e/(1 + e \cos v)]\}[-\cos \Omega_s \cos (\omega + v) + \sin \Omega_s \sin (\omega + v) \cos i] \\
 & + f\{3(\tau - t)[a/\mu(1 - e^2)]^{1/2}(1 + e \cos v) \\
 & + [a^2(1 - e^2)/\mu e][\sin v(2 + e \cos v)/(1 + e \cos v)]\} \times \\
 & [\cos \Omega_s \sin (\omega + v) + \sin \Omega_s \cos (\omega + v) \cos i]. \quad (8)
 \end{aligned}$$

- In these equations  $v$  is the true anomaly and  $\Omega_s$  is related to  $\Omega$  (the longitude of the node measured from the vernal equinox) by  $\Omega_s = \Omega - \lambda$ , where  $\lambda = n_s (t - t_0)$  and  $n_s$  is the mean solar motion about the planet.

In all these equations we need  $v(t)$  to carry out the integrations. This functional relationship comes from Kepler's equation,

$$M \equiv n_p(t - \tau) = \epsilon - e \sin \epsilon \quad (9)$$

where  $M$  is the mean anomaly,  $\epsilon$  the eccentric anomaly,  $n_p$  the mean motion of the particle, and where

$$\tan (v/2) = [(1 + e)/(1 - e)]^{1/2} \tan (\epsilon/2). \quad (10)$$

Kepler's equation may be solved by Newton-Raphson iteration to find  $v(t)$  when  $\tau(t)$  is known,  $\tau(t)$  is found in turn from Eq. (8) if  $v(t)$  is known. Thus (8), (9), and (10) have to be solved as a simultaneous set.

An additional complication is that  $t - \tau$  itself appears in the integrand of Eq. (8). From the law of areas,  $dv/dt = H/r^2$ , we have

$$\begin{aligned} t(v) - \tau &= [a^3(1 - e^2)^3/\mu]^{1/2} \int_0^v dv/(1 + e \cos v)^2 \\ &= [a^3(1 - e^2)^3/\mu]^{1/2} \left\{ -e \sin v/(1 - e^2)(1 + e \cos v) \right. \\ &\quad \left. + 2(1 - e^2)^{-3/2} \tan^{-1}[(1 - e)^{1/2} (1 + e)^{-1/2} \tan v/2] \right\} \end{aligned} \quad (11)$$

To avoid the simultaneous solution of Eqs. (8) [with (11)], (9), and (10), we average Eq. (8) over a cycle and then use  $\langle \tau(t) \rangle$  in lieu of  $\tau(t)$  in solving Kepler's equation. For  $i = 0$  the force terms in the rate equations [Paper I, Eqs. (2), (6), and (7)] simplify to  $\sin(\phi_s + v)$  and  $\cos(\phi_s + v)$ , where  $\phi_s$  is the longitude of perigee from the sun. The mean rate for  $\tau$  is

$$\begin{aligned} \langle d\tau/dt \rangle &= (3a^2 f \cos \phi_s / 2\mu e) [1 + 2e^2 \\ &\quad + \frac{2(1 - e^2)}{\pi} e \int_0^{2\pi} F(v) dv], \end{aligned} \quad (12)$$

where

$$F(v) = \frac{\sin v \tan^{-1} \{[(1 - e)/(1 + e)]^{1/2} \tan v/2\}}{(1 + e \cos v)^2} \quad (13)$$



For small  $e$ , Eq. (12) is comparable to

$$\langle d\phi_s/dt \rangle / n_p = (3a^2 f \cos \phi_s / 2\mu e)(1 - e^2)^{1/2}, \quad (14)$$

where  $n_p$  is the mean rate of motion of a particle about the planet ( $n_p = 2\pi/P = \mu^{1/2}/a^{3/2}$ ). That Eqs. (12) and (14) must agree in the limit of  $e \rightarrow 0$  follows from the consideration that the actual angular rate of particle motion,  $dv/dt$ , is equivalent to  $\Delta\phi_s/\Delta\tau$  and to the mean particle rate,  $n_p$ , as the orbit approaches circularity.

The integral of Eq. (13) may be evaluated by Gaussian integration and then represented as a power series in  $e$ . This procedure gives

$$\begin{aligned} \langle d\tau/dt \rangle = & (3 a^2 f \cos \phi_s / 2\mu e) [1 + 2e^2 \\ & + \frac{4}{\pi}(1 - e^2) e(0.968 + 7.30e - 40.482 e^2 \\ & + 79.0617 e^3 - 43.6806 e^4)]. \end{aligned} \quad (15)$$

Figure 1 illustrates the accuracy of this polynomial representation of  $F(v)$ .

The integrals to obtain the mean rates may be found from the tables of Gradshteyn and Ryzhik (1965, Sect. 2.55). They are of the form

$$I_g^{(n)} \equiv \int_0^{2\pi} dv g(v)/(1 + e \cos v)^n, \quad (16)$$

and the identity

$$I_g^{(n-1)} - e I_g^{(n)} \cos v = I_g^{(n)} \quad (17)$$

is occasionally useful.

## SOLUTION AND CONCLUSION

With  $i = 0$  and  $\langle da/dt \rangle = 0$  and with  $\langle \tau(t) \rangle$  used to solve Kepler's equation for  $v(t)$ , we can readily integrate Eqs. (5), (6), and (7) simultaneously. The results are shown in Figure 2 along with solutions for the mean rates obtained from the equations of Paper I.

The similarity between mean and instantaneous solutions confirms the validity of using the mean rates of change when the orbits are tightly bound to the planet. In paper I [Eq. (18)] the criterion for using means rates was specified as

$$\frac{T_0}{p} = 2.086 \times 10^2 / (a/R_E)^2 \gg 1. \quad (18)$$

In the present case, Eq. (1) gives  $T_0 = 4.7 \times 10^5$  sec = 5.5 days, and we have  $T_0/p = 8.3$ , so that the criterion is crudely satisfied.

The eccentricity for a particle collision with the planet is

$$e_{\text{coll}} = 1 - 1/(a/R_E). \quad (19)$$

From Figure 2 this condition is met at  $e_{\text{coll}} = 0.8$  at the end of 5 orbits or about 3.3 days, which (as expected) is the order of  $T_0$ . The decay of eccentricity shown in Figure 2 is a clear illustration of the orbital instability first noted in Paper I: The longitude of perigee (for a direct orbit) moves asymptotically towards  $90^\circ$ , which is the region where perigee is most rapidly lowered (see Fig. 3). The combined effect is to rapidly vacate orbits in the ecliptic plane (or in any other orbit whose plane contains the Earth-Sun line).

Figures 3, 4, 5, 6, and 7 show sample calculations for loosely bound orbits. In the extreme case of  $a/R_E \gg 1$ , radiation pressure can remove bound orbits within a fraction of an orbit, but in intermediate cases ( $a/R_E \sim 10$ ) the atoms may be either removed or forced into collision with the planet as the eccentricity is increased. Further, the escaping orbits show no overwhelming preference for exit in the anti-sun direction. The existence of a geotail, observed in Ly  $\alpha$ , may be due to charge exchange of loosely bound or escaping atoms with magnetospheric, high-energy protons, followed by a radiation-pressure impulse. However, securely bound atoms (i.e., ones that are only slightly perturbed during one revolution) could not be perturbed sufficiently to cause the observed anti-solar asymmetry to the geocorona.

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## FIGURES

Fig. 1. Polynomial representation of the integral of Eq. (13). The solid curve is

$$[2(1 - e^2)e/\pi] \int_0^{2\pi} F(v)dv$$

and the dashed curve gives  $[4(1 - e^2)e/\pi](0.968 + 7.30 e$

$$- 40.482e^2 + 79.0617e^3 - 43.6806 e^4).$$

Fig. 2. Comparison of the instantaneous elements,  $e$ ,  $\phi_s (= \Omega_s + \omega)$ , and  $\Omega_s$  (dotted curves) with their mean values averaged over an orbit (solid lines) for satellite H atoms in Earth orbit. The initial elements are  $a = 5R_E$ ,  $i = 0$ ,  $e = 0.1$ ,  $\phi_s = 0$ , and  $\Omega_s = 45^\circ$ .

Fig. 3. Orbits of an H atom at  $a/R_E = 5$ . Note that the perigee, initially in the sunward direction, moves to  $\phi_s \approx 90^\circ$  by the fifth orbit. On the sixth orbit, where  $e > 0.8$ , the particles will crash into the planet.

Fig. 4. Progression of the eccentricity for orbits at  $a = 10$  and  $15$  Earth-radii. The initial values are  $e_0 = 0.5$ . These values are actual eccentricities for continuous radiation pressure; the mean elements would be a poor approximation. Quantum accelerations, at mean thrust intervals of 435 sec., produce some dispersion about the continuum acceleration.

Fig. 5. Progression of the perigee longitude from the sun,  $\phi_s$ , for orbits at  $a = 10$  and  $15$  Earth-radii. See legend to Fig. 4.

Fig. 6. Orbit of an H atom about Earth with  $a/R_e = 10$ , obtained from the perturbed elements. The progression of  $e$  and  $\phi_s$  are given in Figs. 4 and 5. This particle collides with the planet on its second orbit. If it missed collision it would escape, but not in the anti-sun direction and not contributing to a geotail of luminescent H.

Fig. 7. Orbit of an H atom about Earth with  $a/R_e = 15$ , obtained from the perturbed elements. The progression of  $e$  and  $\phi_s$  are given in Figs. 4 and 5. This particle escapes on its first orbit, in a direction that does not contribute to a geotail of luminescent H.

















